Engineering Notes

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Skin-Friction Drag at Supersonic-Hypersonic Speeds with Transition

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KNOWLEDGE of the skin-friction drag is of primary importance in the design of advanced aircraft and/or aerospace vehicles. Considerable analytical and experimental studies have been conducted to predict the skin friction for flow conditions that are either laminar or fully turbulent. For mixed flow conditions, the total skin-friction drag coefficient will vary widely depending on the location of the transition point because of the large differences between the drag coefficients for laminar and turbulent flow at identical Reynolds numbers.

When transition from laminar to turbulent flow occurs at low speed, the well-known formula of Prandtl-Schlichting[1] has provided, although empirical in nature, a quick but accurate estimate of the skin-friction drag. Bertram[2] has proposed a semi-empirical method for estimating the skin-friction drag on a delta wing at hypersonic speeds wherein transition is present. However, reviewing his results (Ref. 2, Fig. 15), they do not appear correct. The purpose of this note is to present a method that is similar to Bertram's but which is more rigorous and less complicated and provides numerical results of the skin-friction drag coefficient as a function of the transition on a delta wing.

Consider a flat plate delta wing in a hypersonic flow at small values of angle of attack. Depending on the free-stream Mach number, unit Reynolds number, leading-edge thickness, surface roughness, etc., a portion of the flow on the wing will become turbulent. For a delta wing at small angles of attack, chordwise strip theory is generally valid provided the spanwise pressure gradient induced by the leading-edge-bluntness or boundary-layer-shock interaction is small. It may be noted that ARA has successfully obtained and correlated pressure and force data from a large (48 in. long) 70° swept blunted-leading-edge delta wing at Mach numbers 3 to 8; the data have shown that, except near the nose, chordwise strip theory is a good approximation. Not only is the theoretical treatment of the transition location difficult, but, in addition, its experimental determination is also quite illusive. It is generally accepted that the beginning and end points are determined, respectively, by the minimum and maximum locations of the axial variations of either the surface-pitot-pressure or the heat-transfer rate. There are several thoughts, however, for defining the actual transition location. For instance, some investigators define the location as the point of inflection between the minimum and maximum axial surface-pitot-pressure or heat-transfer rate; others use the end point of the transition region. At moderate supersonic speeds the streamwise distance between the minimum and maximum pressure or heat-transfer rate is relatively small whereas at high supersonic and hypersonic speeds the streamwise distance becomes larger; and also, the minimum and maximum points become increasingly difficult to define. Based on all these considerations, a method for calculating the skin-friction drag with transition becomes quite impractical unless several simplifying assumptions are made. For the present paper, chordwise strip theory is assumed; thus, for a delta wing, the transition from laminar to turbulent flow is represented by a straight line that is parallel to the leading edge. If transition takes place very rapidly, the assumption of a line transition is reasonably justified; for slow transition, the line would then represent the average location in the transition area. Based on these assumptions, the points E and G on the transition lines (GH and EH in Fig. 1) can be used to divide the wing surface into two parts, as follows: 1) the turbulent region and its laminar upstream region (where both are enclosed in the area $ADEGFA$) and 2) the completely laminar regions, DEB and FGC. Assume in the first region that the turbulent flow starts from a virtual leading edge (KM and KN) wherein the total skin-friction drag is the same at the transition line as that of the laminar upstream region; then one can write

$$x_T C_f(x_T) = \bar{x}_T \bar{C}_f(\bar{x}_T) \tag{1}$$

where C_f and $\bar{C}_f$ are the average skin-friction coefficients for laminar and turbulent flow, respectively. Although many sources are available for determining the laminar and turbulent skin-friction coefficients, in order to avoid complexity, the following power-law approximations will be used for the present analysis:

Laminar

$$C_f(x) = A/R(x)^{1/2} \tag{2}$$

Turbulent

$$\bar{C}_f(x) = D/R(x)^{1/q} \tag{3}$$

The constants A, D, and q can be determined from Refs. 3 and 4, respectively, for the laminar and turbulent cases. The Reynolds number based on the distance from the virtual leading edge to the transition line can be obtained by substituting Eqs. (2) and (3) into Eq. (1) to give

$$R(\bar{x}_T) = [A/D\ R(x_T)^{1/2}]^{q/q-1} \tag{4}$$

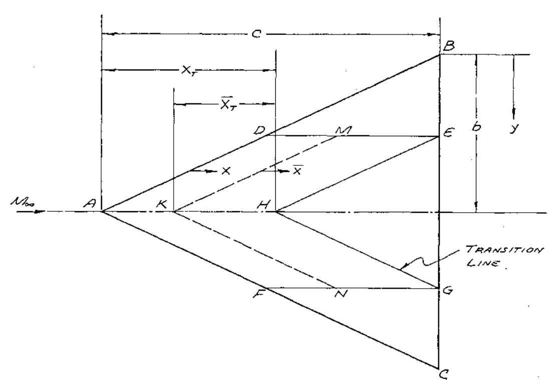

Fig. 1 Delta wing with boundary-layer transition.

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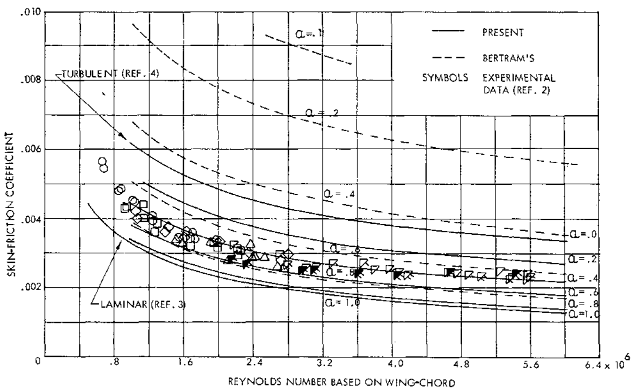

Fig. 2 Skin-friction coefficient of a delta wing with boundary-layer transition at $M_\infty = 6.8$.

where $R(x_T)$ is the Reynolds number based on the downstream distance from the actual leading edge to the transition line. The basic problem is that of determining the location of the transition line. Although analytical methods are not available for predicting the transition line at supersonic Mach numbers, empirical formulas such as that available in Ref. 5 may be used. If such empirical formulas are not applicable, then the transition line should be determined by direct experimental measurements.

The total skin-friction drag coefficient can be evaluated in terms of the coefficients for the two regions defined in Fig. 1 as follows: 1) the mixed flow region defined by the area $AFGED$ and 2) the pure laminar regions defined by the areas DEB and FCG.

As a direct consequence of Eq. (1), the skin-friction coefficient formulas for turbulent flow [Eq. (3)] can be used in the first region, provided the Reynolds number is based on the distance measured from the virtual leading edge to the trailing edge of the delta wing. For the laminar region, Eq. (2) should be used. The expression for the total skin-friction drag coefficient based on the wing area can be written as follows:

$$C_F = \frac{4}{bc}\left\{\int_0^{b/cx_T} C_f(x)xdy + \int_{bc/x_T}^{b} C_f(\bar{x})\bar{x}dy\right\} \tag{5}$$

where $x = (c/b)y$ and $\bar{x} = (c/b)y + (\bar{x}_T - x_T)$.

Equation (5) can be written in dimensionless form by using the wing-chord as a reference length. Let

$$\xi = x/c \qquad \bar{\xi} = \bar{x}/c \tag{6}$$

and express the transition distance x_T as a percentage of the wing-chord,

$$a = x_T/c \tag{7}$$

Equation (5) becomes

$$C_F = 4\left\{\int_0^a C_f(c\xi)\xi d\xi + \int_a^1 \bar{C}_f(c\bar{\xi})\bar{\xi}d\bar{\xi}\right\} \tag{8}$$

where

$$\bar{\xi} = \xi + \{[R(x_T)/R(c)] - a\} \tag{9}$$

Equation (8) can be integrated in closed form by using Eq. (9) and the expressions for C_f and $\bar{C}_f$ given, respectively, by Eqs. (2) and (3). This result is provided as follows:

$$C_F = 4\left\{\frac{2A}{3R(c)^{1/2}}a^{3/2} + \frac{qD}{(2q-1)R(c)^{1/q}} \times \left[\left(1 + \frac{R(\bar{x}_T)}{R(c)} - a\right)^{(2q-1)/q} - \left(\frac{R(\bar{x}_T)}{R(c)}\right)^{(2q-1)/q}\right]\right\} \tag{10}$$

wherein the expression for $R(\bar{x}_T)/R(c)$ can also be written in terms of the parameter a as follows:

$$\frac{R(\bar{x}_T)}{R(c)} = \frac{[(A/D)a^{1/2}R(c)^{1/2}]^{q/(q-1)}}{R(c)} \tag{11}$$

Based on Eqs. (10) and (11), the skin-friction coefficient at a given Mach number can be determined as a function of Reynolds number once the location of the transition as a fraction of wing-chord a is known. For the purposes of comparison with Bertram's results,[2] Eq. (10) was evaluated at $M = 6.8$ wherein values of A, D, and q (given in Ref. 2) were used, i.e., $A = 1.22$, $D = 0.243$, and $q = 3.03$. These results are plotted in Fig. 2 as a function of the Reynolds number based on wing-chord, and the transition parameter a. Also shown are Bertram's results together with the experimental data that was also available in Ref. 2, Fig. 15. Comparison of Fig. 2 indicates that although the results for the laminar case, $a = 1.0$, agree with Bertram, the results for the turbulent region, $a = 0$, do not agree. The turbulent skin-friction drag based on the present paper does agree, however, with the results given by Van Driest.[4] Consequently, when the experimental data is compared with the theoretical curves for various values of a in Fig. 2, the interpretation of the data with respect to the amount of turbulent and laminar flow that exists is subject to question. For example, if the experimental data is compared with Bertram's curves in Fig. 2, the value of a would be from $a = 0.65$ to 0.75 which indicates that the flow on the wing is primarily laminar. On the other hand, if curves of the present paper are used, values of $a = 0.35$ to 0.45 are obtained which indicate that the flow on the wing is primarily turbulent.

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A 20-Ft-Diam Ribbon Parachute for Deployment at Dynamic Pressures above 400 PSF

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Introduction

In order to satisfy the continual need for recovery systems capable of deployment at higher dynamic pressures, a parachute research program has been carried on over the last ten years to increase the capability of ribbon parachutes from

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